# Growth Rates

A simple model of population growth would add the individuals born and those that immigrated into the population during a given time period (such as a year) and subtract those who died and those who emigrated during the same time period to the numbers that were present for the population at the beginning of the time period (such as the beginning of the year.) An equation to represent this model is below:

$$P_2 = P_1 + (B-D) + (I-E)$$

where, P2 is the population at time 2
P1 is the population at time 1
B is the number of births between time 2 and time 1
D is the number of deaths between time 2 and time 1
I is the number of immigrants between time 2 and time 1
B is the number of emigrants between time 2 and time 1

Allowing ΔP to represent P2-P1, Δt to represent the time interval of interest, and ignoring migration, we get,

$$\Delta P = (B-D)\ \Delta t$$

A more useful form of the above equation counts births and deaths as a fraction of a population over the time interval, leading to the following equation:

$$\Delta P = (b-d)\ \Delta t$$

where, b = B/P
d = D/P

Usually b-d is given the symbol r and is called the *intrinsic or natural rate of increase* (or sometimes just the *rate of increase*.) Often b and d are given as a percentage of the population. rather than as a fraction. The difference between b and d then becomes the *percentage rate of increase*. A little calculus (but more than you need to know) yields the following relationship:

$$P = P_0\ e^{(b-d)t} \text{ or } P = P_0\ e^{rt}$$

This equation allows us to calculate the population size P at any given time (usually in years, but not necessarily so) as long as we know either the intrinsic rate of increase or the birth and death rates for a population.

An interesting form of the equation results when P is twice $P_0$.

$$P = 2P_0 = P_0\ e^{rt}$$

A little more mathematical wizardry gives the following useful equation:

$$t = \ln 2 / r, \text{ or } t = 0.693/r$$

This equation allows us to calculate the doubling time for any rate of increase (including savings bank interest rates - for percentage rates, the equation is t = 69.3/r). You will use the equations above to learn some things about simple growth models. You can calculate the rate of increase from the following form of this same equation:

$$r = (\ln P_2 - \ln P_1) / t$$

In this equation, you need to have two population sizes ($P_2$ and $P_1$) and the time interval between them. To get r as a percentage (%), simply multiply by 100%. The symbol ln stands for the natural log and can easily be obtained with most calculators.

## Part 1. Doubling time calculations.

The table below has estimates for the size of the human population at various stages of history (and prehistory). Determine the % growth rate and doubling time.

| Period (years) | Population | R (in % per year) | Doubling time |
|---|---|---|---|
| 300,000 BC | 1 million | | |
| 10,000 BC | 3 million | | |
| 1 AD | 200 million | | |
| 1650 AD | 0.5 billion | | |
| 1900 AD | 1.6 billion | | |
| 1950 AD | 2.4 billion | | |
| 2000 AD | 6.0 billion | | |

## Part 2. Comparison of initial population and rate of increase.

Equipment: scientific calculator, graph paper

In the table below, you will develop several population growth histories. Then graph them. Use a linear scale for your graph.

| Year | R=1<br>P=100 | R=1<br>P=200 | R=1<br>P=1000 | R=2<br>P=100 | R=10<br>P=100 |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |
| 10 | | | | | |
| 20 | | | | | |
| 30 | | | | | |
| 40 | | | | | |
| 50 | | | | | |

## Part 3. Conclusions

1. What difficulty would you have in graphing the size of the human population from the data given in this lab?

2. What lessons (if any) do the historical data you have analyzed and the comparison of data on how the intrinsic growth rate affects population growth have for the future of human growth?

3. Compare the effects on population growth of doubling the intrinsic rate of increase to doubling the initial population size. Use features of the graphs you have drawn in your comparison.